

Non-BPS excitations of D-branes and black holes*

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This note discusses some results on the non-BPS excitations of D-branes. We show that the excitation spectrum of a bound state of D-strings changes character when the length of the wrapping circle becomes less than $\sim g^{-1}L^{(S)}$. We review the observed relation between the low energy absorption cross-section of D-branes and the low energy absorption cross-section for black holes. We discuss various issues related to the information question for black holes.

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This is an expanded version of a talk given at Strings '96, Santa Barbara, with the title 'Comparing decay rates for D-branes and black holes'. Some results on the non-BPS spectra of higher branes have been extended to cover the case of 0-branes, given the interest in 0-branes in this conference. Some earlier work on the black hole information issue is reviewed as well.

1. Introduction

Recently there has been an extensive and fruitful investigation into the count of the BPS states in string theory when some chosen charges of the configuration are held fixed. Following suggestions of Susskind [1], Russo and Susskind[2] and Vafa[3], Sen [4] computed the logarithm of the number of BPS states of the heterotic string with a fixed total charge, and found this to equal the area of the stretched horizon of the corresponding black hole (measured in planck units), upto a constant of order unity. The advent of D-branes [5] provided objects in string theory carrying a variety of different charges. Strominger and Vafa [6] constructed a black hole model in 4+1 dimensions carrying three different charges and thus possessing a nonsingular horizon. They found that the entropy for branes with a given set of charges *exactly* equalled the Bekenstein-Hawking entropy for a black hole carrying the same charges. Other examples of this correspondence were soon developed [7][8]. These results suggest that the Bekenstein entropy defined by the area of the horizon is in some way a count of the possible microstates of the black hole, though it is not yet clear where these microstates reside. They also provides a striking validation of string theory, with the large number of perturbative and non-perturbative particle species in the theory finding a natural place in reproducing the Bekenstein-Hawking entropy predicted by classical gravity and quantum field theory.

To investigate issues of Hawking radiation and information loss, we must consider non-BPS states of the theory, since extremal holes do not radiate. When investigating the physics in this domain one has to be more careful, since we do not have the non-renormalisation theorems that applied to the case of BPS-states. But there are several results [9][10][11][12][13] that encourage the belief that the physics captured by the regime of non-BPS D-brane physics where we are able to do computations, is in some way related to the physics of black holes, at least at low energies.

In this note we do the following:

(a) We examine the spectrum of a bound state of 0-branes, when the spacetime has been compactified on a circle. The spectrum exhibits some curious features when the scale of compactification becomes smaller than the natural size of the 0-brane bound state. The spectrum in this domain is found by using dualities on the known spectrum of the elementary string, following the methods in [13].

(b) We turn the above process around, starting from the spectrum of the bound state of n_w 0-branes when the compactification scale is large (essentially noncompact spacetime) and obtaining the excitations for the bound state of n_w D-strings when the length of the wrapping circle is smaller than $L_{\text{cr}} \equiv g^{-1}L^{(S)}$. (Here $g = e^\phi$ is the elementary string coupling and $L^{(S)} = (\alpha')^{1/2}$ is the length scale of the elementary string.) We thus find that the lowest lying excitations for the D-string change character as the length of the wrapping circle L drops below L_{cr} : for $L \gg L_{\text{cr}}$ we have the universal spectrum of vibrations of a single string of length $n_w L$, while for $L < L_{\text{cr}}$ we get position independent oscillations coming from the nonzero thickness of the bound state.

(c) We review the computation for the absorption cross section of low energy quanta into a combination of branes. The branes are chosen to carry the charges of a black hole with nonzero horizon area in 4+1 dimensions, following [6][9]. The absorption cross section of the quanta studied agrees with the low energy absorption cross section for the corresponding black hole.

(d) We discuss the black hole information paradox. In particular we discuss the large quantum gravity effects that appear to exist in some approaches [14][15] to perform the full calculation of Hawking radiation, and argue that these are only an apparent effect arising from a turning point in a semiclassical quantum gravity wavefunction [16]. We review the breakdown of the classical field limit in 1+1 dimensional string theory before the threshold of black hole formation [17]. We discuss the possible implications of the results on the cross section agreements between D-branes and black holes, and the issue of measurability of black hole hair.

2. Non-BPS spectrum of 0-branes and 1-branes.

Let the spacetime be flat Minkowski $M^9 \times S^1$, where the coordinate X^9 has been compactified on a circle. Let $g = e^\phi$ be the elementary string coupling. Let $L^{(S)}$ be the length scale associated with the elementary string:

$$L^{(S)} = (2\pi)^{1/2}(T^{(S)})^{-1/2} \quad (2.1)$$

and $L^{(D)}$ be the length scale associated with the D-string of type IIB string theory:

$$L^{(D)} = (2\pi)^{1/2} (T^{(D)})^{-1/2}, \quad T^{(D)} = T^{(S)}/g \quad (2.2)$$

Let the D-string have winding number n_w around X^9 , and no momentum along X^9 . Let the length of the compactified circle be

$$L = AL^{(D)} = AL^{(S)}g^{1/2} \quad (2.3)$$

If we T-dualise in the compact direction X^9 , the new length of the circle will be

$$L' = A^{-1}g^{-1/2}L^{(S)} \quad (2.4)$$

and the new coupling will be

$$g' = g[L'/L]^{1/2} = g[A^{-2}g^{-1}]^{1/2} = g^{1/2}A^{-1} \quad (2.5)$$

The D-string of type IIB string theory will change to a bound state of n_w 0-branes of type IIA theory, with no momentum in the X^9 direction.

2.1. Spectra

We know the following about the spectrum of the elementary string. Suppose the radius of the X^9 circle is

$$L_1 = AL^{(S)} \quad (2.6)$$

Let the elementary string have winding number n_w around X^9 , and no momentum along X^9 . If $g = e^\phi \ll 1$, and $A > 1$, then we have a spectrum of long lived excitations for the low lying states, given by essentially the free string spectrum. The 9-dimensional masses of the string states are

$$m^2 = (n_w L_1 T^{(S)})^2 + 8\pi T^{(S)} N \quad (2.7)$$

where N is the excitation level over the ground state in both the right and left sectors (which can each be either Ramond (R) or Neveu-Schwarz (NS)). The term ‘long lived excitation’ used above stands for the fact that the lifetime of the excited state with excitation energy ΔE is much larger than $(\Delta E)^{-1}$. The restriction on A may be relaxed somewhat, but we cannot let A become too small for nonzero g , for then the spectrum changes, as we will discuss later. The restriction $A > 1$ is a convenient starting point for our present purposes. The reason for the change of spectrum is that if there is a very small compactified

circle, then there is a very low mass winding state that can contribute in loop corrections to the string eigenstate.

By S-duality of the type IIB theory, we can conclude the following for the spectrum of the D-string. Suppose we have $g_D = g^{-1} \ll 1$, and the length of the X^9 circle is

$$L = AL^{(D)} = AL^{(S)}g^{1/2} \quad (2.8)$$

and $A > 1$. Then we have a spectrum of low lying long lived excitations given by

$$m^2 = (n_w LT^{(D)})^2 + 8\pi T^{(D)} N \quad (2.9)$$

For n_w , A , fixed, and $N \gg n_w^2 A^2$, we get

$$m \approx (T^{(D)})^{1/2} \sqrt{8\pi} \sqrt{N} \quad (2.10)$$

For n_w , N fixed, and $A \gg n_w^{-1} \sqrt{N}$, we get

$$m \approx (T^{(D)})^{1/2} \sqrt{2\pi} n_w A + (T^{(D)})^{1/2} \frac{2\sqrt{2\pi}}{n_w A} N \quad (2.11)$$

On T-dualisation we get the 0-brane state with the same energy levels. Expressed in terms of g' rather than g the energy levels are for n_w , A , fixed and $N \gg n_w^2 A^2$:

$$m \approx (T^{(S)})^{1/2} \sqrt{8\pi} (g')^{-1} A^{-1} \sqrt{N} \quad (2.12)$$

and for n_w , N fixed and $A \gg n_w^{-1} \sqrt{N}$:

$$m \approx T^{(D)}_0 n_w + (T^{(S)})^{1/2} (g')^{-1} \frac{2\sqrt{2\pi}}{n_w A^2} N \quad (2.13)$$

where

$$T^{(D)}_0 = T^{(S)} (g')^{-1} L^{(S)} = (T^{(S)})^{1/2} (g')^{-1} \sqrt{2\pi} \quad (2.14)$$

is the tension (i.e. the mass) of the 0-brane.

Now we discuss what we know directly from the spectrum of bound states for the 0-branes. Let

$$L_0 \equiv (g')^{1/3} L^{(S)} \quad (2.15)$$

In [18][19][20] it was argued that for

$$g' \ll 1 \quad L' \gg L_0, \quad (2.16)$$

the length scale of the bound state of $n_w > 1$ zero branes is $\sim L_0$. Further it was argued that the low lying non-BPS excitations (carrying no net charge) are given by levels with spacing of order

$$(\Delta E)_0 \sim (g')^{1/3} (T^{(S)})^{1/2} \quad (2.17)$$

It is not immediately clear, however, that there is any discernable level structure in the low level excitations of the zero brane bound state. The argument of [18] used a separation of slow and fast modes to do a semiclassical expansion; there is however no small parameter that actually governs this separation. There may exist broad resonances with width of the same order as the height, exhibiting structure at the scale (2.17), but there is no clear evidence for this either. It is true, however, on dimensional grounds, that the only scale exhibited by the excitations is that given by (2.17).

For later use we note that

$$L'/L_0 = A^{-2} (g')^{-4/3} = A^{-2/3} g^{-2/3}. \quad (2.18)$$

2.2. Elementary string \rightarrow D-string \rightarrow 0-branes

The spectrum (2.9) for the D-string was obtained for $g \gg 1$, $A > 1$. From (2.4) we have that $L' \ll L^{(S)}$. Using (2.5) we see that we can choose $A \sim 1$ to get $g' \gg 1$, or choose A sufficiently large so that we have $g' \ll 1$. Let us make the latter choice. Then since the spectrum does not alter under T-duality, we find from (2.13) that for the type IIA theory 0-brane bound state we have long lived excitations with separation

$$(\Delta E)_1 = \frac{2L'}{n_w} T^{(S)} \quad (2.19)$$

The spectrum (2.19) is very different from (2.17). In obtaining (2.19) we have used a parameter range where

$$L'/L_0 = A^{-2/3} g^{-2/3} \ll 1 \quad (2.20)$$

where the inequality follows because $A > 1$ and $g \gg 1$. Thus the 0-brane state has been ‘squashed’ in the compact direction to a size much smaller than the natural scale of 0-brane bound states.

We can use this result to speculate on the structure of 0-brane bound states. The scale of excitations (2.17) can be understood heuristically in the following way. For a bound state of two 0-branes, say, we get an excited state by attaching a pair of open strings with the opposite orientation, beginning at one 0-brane and ending at another. If the 0-branes

are a distance l apart, then the energy from the tension of these strings is $V \sim lT^{(S)}$. On the other hand confining the 0-branes within a region of size $\sim l$ gives a kinetic energy for each 0-brane $K \approx p^2/(2M) \sim l^{-2}(T^{(S)})^{-1/2}g'$ (M is the mass of the 0-brane). Minimising the total energy of excitation $(\Delta E)_0 = V + T$ gives $l \sim (T^{(S)})^{-1/2}(g')^{1/3}$, and $(\Delta E)_0 \sim (T^{(S)})^{1/2}(g')^{1/3}$, in accordance with (2.17).

If we were doing the quantum mechanics of two pointlike objects, after compactification of X^9 to a small circle the argument of the preceeding paragraph would still apply, and again yield the scale (2.17). The wavefunctions would simply reduce to constants in the compact direction. But if the average separation between the 0-branes is $\sim L_0$, then the open strings stretching from one 0-brane to the other would give an energy scale $\sim L_0 T^{(S)}$ which is much larger than (2.17). If we start and end the open strings on the same zero brane, while wrapping it on the compact circle X^9 , then we get the energy levels

$$(\Delta E)_2 = 2NL'T^{(S)} \quad (2.21)$$

which differs from (2.19) by the factor n_w .

What physical picture can give the extra n_w in (2.19)? Since we have kept $g' \ll 1$, it is tempting to look for a picture of the excitation in terms of a small number of open strings, though this might be invalid due to loop corrections in the presence of the very small compactification scale. We list three possibilities:

(1) We must use fractional open strings, with tension $(n_w)^{-1}T^{(S)}$ in (2.21), following the notion of fractional branes discussed in [21].

(2) The natural scale of the 0-brane bound state is $\sim L_0$ in noncompact space, but if one direction is compactified to a length much smaller than L_0 then the bound state becomes reduced to that compactification scale in all directions. If this happens then the open strings stretching from one 0-brane to another may yield levels of the order (2.19), though there is no immediate reason for this precise form.

(3) The 0-branes in the compactified spacetime have a disclike shape, (with perhaps the scale L_0 in the noncompact direction), and these discs are stacked parallel to each other with separations L'/n_w along the compact direction X^9 . The open strings can stretch from one disc to the next one, starting at any point on the first disc and stretched parallel to the direction X^9 . The wavefunction of the open string is a uniform superposition over the various possible locations of the end point on the first disc. (In fact to get a correct count of BPS states it appears more natural to use open strings with fractional tension here just

as in (1) above, with the minimum excitation involving fractional open strings that stretch from each 0-brane to the next.)

If possibility (3) is correct then it could be interesting for the following reason. D-brane excitations at weak coupling (and no small compactified directions) are given by open strings that are attached to a hyperplanes that are infinitely thin in the Dirichlet directions. But the ideas of Susskind about black holes suggest that at strong coupling the D-branes should be described by an effective theory that has open strings ending on an extended surface (the horizon) which is not itself the surfaces of the D-branes that the black hole was constructed with. The description (3) above also requires an effective extended endpoint for the open strings attached to a 0-brane.

2.3. 0-branes \rightarrow D-strings.

Let us now start from the other side, with a bound state of $n_w > 1$ 0-branes, in a domain of parameters where we know something about the spectrum:

$$g' \ll 1, \quad L'/L_0 \gg 1 \quad (2.22)$$

Then as mentioned above, the spectrum has structure at the energy scale (2.17), and this will be also the structure of the spectrum of any string or brane obtained through dualities. From (2.18), we have

$$A \ll (g)^{-1} \quad (2.23)$$

$$g = (g')^2 A^2 \ll (g')^{2/3} \ll 1 \quad (2.24)$$

$$L = AL^{(D)} \ll g^{-1} L^{(D)} = L^{(D)} g_D = g^{-1/2} L^{(S)} \quad (2.25)$$

Thus we have weak elementary string coupling g and the length L of the D-string much longer than the elementary string scale $L^{(S)}$. At first we might expect that in this situation we would get the spectrum of excitations given by attaching open strings to the D-strings. If the n_w D-string bound state implies just the naive n_w valued Chan-Paton factors at the ends of the open strings then the spectrum would be (for no net momentum in the X^9 direction)

$$E_N = \frac{4\pi N}{L} = \frac{2\sqrt{2\pi}N}{A} g^{-1/2} T^{(S)1/2} \quad (2.26)$$

with degeneracy n_w^2 for each level. If the n_w D-strings behave as one string of length $n_w L$ then the spectrum would be

$$E_N = \frac{4\pi N}{n_w L} = \frac{2\sqrt{2\pi}N}{n_w A} g^{-1/2} T^{(S)1/2} \quad (2.27)$$

with degeneracy unity for each level.

What we actually have from (2.17) by duality is structure in the energy spectrum at the scale

$$(\Delta E)_0 \sim g^{1/6} A^{-1/3} T^{(S)1/2} \quad (2.28)$$

From (2.23), we see that the range of validity of our analysis is $A < g^{-1}$. For $A \sim g^{-1}$, n_w small, all the scales (2.26), (2.27), (2.28) are $\sim g^{1/2} T^{(S)1/2}$. But as we reduce A below $\sim g^{-1}$, the levels (2.26), (2.27) become higher than the scale (2.28) at which we first see the structure of excitations.

The above observation may be relevant to the consideration of non-BPS entropy and absorption coefficients of D-brane configurations that are anticipated to resemble black holes. The low energy spectrum used in [9] was analogous to (2.26), and that used in [21] was analogous to (2.27). But as we reduce the length of the D-string, which happens as we reduce the compactification scale to go towards a black hole, we find that excitations at the scale (2.28) dominate the low energy physics.

At an intuitive level, the appearance of the scale (2.28) might be understood as follows. If the D-string is very long (longer than $\sim g^{-1} L^{(S)}$) then the thickness of the strands making up the string is less than the typical separation between the strands in the process of oscillation. Thus we simply get the universal spectrum of one string of length $n_w L$. But for the D-string shorter than $\sim g^{-1} L^{(S)}$ the ‘breather modes’ of the thick soliton are of lower energy than the universal oscillation modes of the string, and dominate the low energy excitations. The timescale of these latter oscillations are probably the same as the timescale for dissociation of the bound state, so it is not clear if these should be thought of as oscillations at all.

Using S-duality we can state the result corresponding to (2.28) for the elementary string. If we take an elementary string at large coupling $g \gg 1$, wound on a circle of length smaller than $gL^{(S)}$, then we will get an excitation spectrum that has structure at scale

$$\Delta E \sim g^{-2/3} A^{-1/3} (T^{(S)})^{1/2} \quad (2.29)$$

3. Absorption into D-branes

We consider the absorption of low energy quanta into extremal black holes in 4+1 spacetime dimensions, and compare this to the absorption by D-branes carrying the same charges as the black hole [12][13]. Let the spacetime be $M^6 \times T^5$, where the directions

$X^5 \dots X^9$ have been compactified on the torus T^5 . The black hole must carry three nonzero charges in order to have a classically nonvanishing area of the horizon. Following [6][9] we make the corresponding D-brane configuration by taking one D-5-brane wrapped on the torus, a D-string wrapped n_w times around the X^9 cycle and bound to this D-5-brane, and let the D-string carry momentum along the X^9 direction.

A D-string of length L may be considered as a system with some discrete energy levels with spacing ΔE which is independent of E . Consider an initial state at $t = 0$ where the D-brane system is in its BPS ground state and a massless closed string state of energy k_0 is incident on it. Let the amplitude to excite the D-string to any one of the excited levels per unit time be R . (For t large, only the levels in a narrow band will contribute, and in this band we can use the same R for each level.) Then the amplitude that the system in an excited state with energy E_n at a given time t is given by

$$A(t) = R e^{-iE_n t} \int_0^t dt' e^{i(E_n - k_0)t'} = R e^{-\frac{i}{2}(E_n + k_0)t} \left[\frac{2 \sin[(E_n - k_0)t/2]}{(E_n - k_0)} \right] \quad (3.1)$$

The total number of quanta absorbed in time t is thus given by

$$P(t) = \sum_n |R|^2 \left[\frac{2 \sin[(E_n - k_0)t/2]}{(E_n - k_0)} \right]^2 \rho(k_0) \quad (3.2)$$

where $\rho(k_0)$ denotes the occupation density of state of the incoming quantum. For large length of the D-string L we can replace the sum by an integral

$$\sum_n \rightarrow \int \frac{dE}{\Delta E} \quad (3.3)$$

in which case the rate of absorption $\mathcal{R}_A = P(t)/t$ evaluates to

$$\mathcal{R}_A(t) = \frac{2\pi |R|^2}{\Delta E} \rho(k_0) \quad (3.4)$$

For our case of the D-string on the 5-brane,

$$\Delta E = \frac{4\pi}{n_w L} \quad (3.5)$$

Here we have used the fact that for a sufficiently large wrapping radius or sufficiently large g a bound state of D-strings exhibits the excitation spectrum of a single multi-wound string of length $n_w L$ [22].

Consider the absorption of a quantum of the 10-dimensional graviton h_{12} , with no momentum or winding along the compact directions. This is a neutral massless scalar of the 5-dimensional theory. There are two open string states that can be created on the D-string in absorbing this graviton. We can have the string with polarisation 1 travelling left on the D-string and the open string with polarisation 2 travelling right, or we can have the polarisations the other way round. This means that there are two series of closely spaced levels that will do the absorption, and so the final rate of absorption computed from (3.4) will have to be doubled.

To find R , we have to examine the action for the D-string coupled to gravity. Writing the action with only the fields that we will use below:

$$S = \frac{1}{2\kappa^2} \int d^{10}X [R - \frac{1}{2}(\partial\phi)^2] + T \int d^2\xi e^{-\phi/2} \sqrt{\det[G_{mn}]} \quad (3.6)$$

where

$$G_{mn} = G_{\mu\nu}(X) \partial_m X^\mu \partial_n X^\nu \quad (3.7)$$

and T is a tension related to the tension of the elementary string by $T^{(S)} = e^{\phi/2}T$. Note that the tension of the D-string is $T^{(D)} = Te^{-\phi/2}$. Expanding this action to lowest required order, with $G_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$:

$$S \rightarrow \int d^{10}X \frac{1}{2}(\partial h_{ij})(\partial h^{ij}) + \frac{1}{2}(\delta_{ij} + 2\kappa h_{ij}) \partial_\alpha (\sqrt{T^{(D)}} X)^i \partial^\alpha (\sqrt{T^{(D)}} X)^j \quad (3.8)$$

From (3.8) we find for the amplitude per unit time for the graviton to create any one of these two possible open string configurations to be

$$R = \sqrt{2}\kappa |p_1| \frac{1}{\sqrt{2k^0}} \frac{1}{\sqrt{L}} \frac{1}{\sqrt{V_c}} \frac{1}{\sqrt{V_T}} \rho_L^{(1/2)}(|p_1|) \quad (3.9)$$

p_1 is the momentum of the massless open string travelling left, say, while k^0 is the energy of the absorbed quantum. Here we have separated the term $\frac{1}{\sqrt{V}}$ into contributions from the string direction X^9 , the remaining four compact directions (denoted by the subscript c) and the transverse noncompact spatial directions (denoted by the subscript T). We have also included the term

$$[\rho_L(|p_1|)]^{1/2} = \left[\frac{T_L}{|p_1|}\right]^{(1/2)}, \quad T_L = \frac{S_L}{\pi n_w L} \quad (3.10)$$

which gives the Bose enhancement factor due to the population of left moving open string states on the D-string [9]. Here S_L is the entropy of the extremal configuration, given by the count of the possible ways to distribute the N quanta of momentum among different left moving vibrations of the D-string:

$$S_L = 2\pi\sqrt{n_w N} \quad (3.11)$$

and equals the Bekenstein entropy of the black hole with the same charges as the D-brane configuration.

The absorption cross section is given by

$$\sigma = 2\mathcal{R}_A/\mathcal{F} \quad (3.12)$$

where $\mathcal{F} = \rho(k_0)V_T^{-1}$ is the flux, and the factor of 2 was explained before eq. (3.9).

Note that

$$\frac{\kappa^2}{LV_c} = 8\pi G_N^5 \quad (3.13)$$

and that for the given choice of momenta

$$k_0 = 2|p_1| \quad (3.14)$$

Then we find

$$\sigma = A \quad (3.15)$$

where $A = 8\pi G_N^5 \sqrt{n_w N}$ is the area of the extremal black hole with one 5-D-brane, n_w windings of the 1-D-brane, and momentum charge N .

To compare this to the classical absorption cross section at low energies, one solves the wave equation for the incoming quantum in the metric of the black hole (using the approximation that the wavelength is much larger than the Schwarzschild radius of the black hole). Such a calculation for the 4+1 dimensional hole is performed in [12], following the calculation for 3+1 dimensions in [23]. The result is precisely (3.15). Thus we get agreement between the absorption cross-sections of the D-branes on the one hand, and the black hole they will form for a different choice of coupling on the other.

In [13] it was shown that the cross section for the dilaton agrees as well between the black hole and the D-brane configuration. Note that the D-string can only oscillate within the 5-D-brane, which is wrapped on the internal directions. Thus at this order of calculation, only 5-dimensional scalars are absorbed, and vectors and gravitons have no

absorption cross-section. This agrees with the fact that for 3+1 dimensional holes the low energy cross section vanishes for vectors and gravitons [24]. By supersymmetry, we expect that the classical cross section for spin 1/2 quanta is related to that for scalars, and is thus $\sim A$ as well. The D-brane calculation yields the order $\sim A$ also, since we can have a fermionic open string as the right mover and the bosonic open string again as the left mover, giving a spin-1/2 quantum in the emitted state.

In the above calculation we observe that the cross section for low energy massless neutral scalars was precisely the horizon area, in 5 dimensions. This is also the case in 4 dimensions, so one wonders if for such quanta one always gets the area of the horizon, and if there are further universalities among the low energy cross sections for particles with various spin. This issue is addressed in [25], where it is shown that such is indeed the case.

Recently there has been interesting progress on this issue. The above calculation for neutral scalars has been extended to charged quanta in both 4 and 5 dimensions [26]. Further, it was shown in [27] that the D-brane configuration reproduces the characteristics of black hole grey-body factors for both neutral and charged quanta.

4. Black hole information

4.1. Quantum gravity effects

It has been often felt that the calculation of Hawking that gives Hawking radiation in an essentially thermal form should suffer from quantum gravity corrections that might permit information to leak out with the radiation. This possibility has been stressed by 't Hooft, and a calculation was performed in [14] which indicated that commutators of operators associated to the radiation in a full theory of quantum gravity became large at the horizon. Similar large effects were found in [15] in a Hamiltonian formalism.

Should we consider this as convincing evidence that the information paradox is in fact just an effect of using semiclassical gravity where quantum gravity had to be used? In [16] it was argued that such need not be the case. In investigating the issue of unitarity, it is best to work with states rather than operators, since if one uses operators there still remains the question of which states the operators must be sandwiched between to decide if large quantum gravity effects exist.

In the Hamiltonian formulation of quantum gravity (as opposed to the path-integral formulation) the wavefunction is a function only of spacelike 3-geometries and matter on the 3-geometries (for the case of 3+1 dimensional spacetime physics). The time direction

arises as the phase of the wavefunction of the 3-geometries, in a WKB approximation of this wavefunction [28]. If the matter density is small compared to planck density, we have a Born-Oppenheimer approximation where the ‘fast’ variable is the metric and the ‘slow’ variable is the matter (i.e. the phase of the variable representing gravity oscillates much faster than the phase of the matter variable). The quantum matter obeys a Schrodinger equation in the ‘time’ that emerges in this WKB approximation.

But if spacetime is obtained in this way, then we have the issue of what happens when the fast variable (the parameter labelling the 3-geometries) encounters a turning point. At a turning point the fast variable would not be fast any more, and we can expect that the semiclassical description will be invalid. As the gravity variable recedes from the turning point and becomes ‘fast’ again, the semiclassical gravity description is valid again, but we can ask for the total ‘damage’ created by the turning point: namely the error created by using the semiclassical description through the turning point where it was not really valid.

This ‘damage’ was computed in several simple examples in [16], and it was found that while the semiclassical description indeed breaks down in the vicinity of the turning point, the departure from semiclassicality erases itself as we recede from the turning point, at least in simple models of quantum gravity, leaving a small but computable net effect of the temporary loss of semiclassicality. This ‘miraculous’ cancellation of large departures from semiclassicality can be traced to the fact that by a canonical transformation a turning point can be relocated or removed.

In the black hole context the large temporary departures from semiclassicality appear as large quantum gravity effects in the Schwarzschild coordinate near the horizon. It is plausible that these large effects are closely related to the large commutator effects seen in [14]. Thus we conclude that large quantum gravity effects may be only apparent effects and not real effects, at least in simple models of quantum gravity which do not involve extended objects like strings or branes.

4.2. 1+1 dimensional noncritical string theory

String theory provides a renormalisable model of quantum gravity, so one would like to investigate black hole information issues using string theory. Computing just a few orders of perturbation theory, however, is unlikely to yield insights on this problem, beyond what is known from semiclassical gravity. Luckily it is possible to perform a sum over all string loop diagrams in the case of the noncritical 1+1 dimensional string. As shown in [29] the $c = 1$ random surface model can be recast as a theory of free fermions, where the fermi

surface profile acts like the gradient of a scalar field defined over a 1-dimensional spacelike dimension, with time being the second dimension.

As observed by Polchinski [30], it is possible for the fermi surface to form ‘folds’ in the process of evolution, which destroys the immediate relation between the local fermi level and the value of the scalar field. In particular this effect occurs before the threshold of ‘black hole formation’ in this theory. We can take a coherent pulse made from the quanta of the scalar field, and have this pulse move in from infinity towards the strong coupling ‘wall’ present in this model in the vacuum. For low amplitudes of the incident pulse, a slightly distorted but still coherent pulse returns after reflection from the wall. But as the amplitude of the initial pulse exceeds a certain threshold, the returning pulse develops a ‘fold’ in the fermi surface, in the description through free fermions. What does this mean in terms of the scalar field description?

In [17] it was shown that the scalar field state after fold formation corresponds to a collection of incoherent quanta, with frequencies that range from low values to very high values: the average frequency amplification over the frequency of the initial pulse is of the order of the square root of the number of quanta in the initial state. In particular this is not the profile expected of thermal radiation that may result from a process of virtual black hole formation and evaporation. It appears that as the initial pulse enters a strong coupling area, the approximation that the string theory is a theory of a single particle species (the tachyon) breaks down, and stringy effects create an outgoing state of a form that cannot arise in a field theory with just the tachyon field. It would be interesting if this phenomenon were to happen in higher dimension string theories as well, as we approach the threshold of black hole formation.

4.3. Scattering off black holes

At least for extremal and near extremal black holes, there is now a fairly convincing case that the Bekenstein entropy should be interpreted as some count of possible microstates. Can we scatter quanta off a black hole, and in the process determine which state the black hole is in? This would indicate that black holes are not really ‘black’, and are much like ordinary particles. We have seen in the last section that the absorption cross section of D-branes matches that of black holes at low energies. But with the D-branes, if we do scattering experiments we would indeed know which microstate the branes were in. For example, suppose we examine the absorption of a 5 dimensional scalar of energy $\frac{4\pi N}{n_w L}$, in the notation of the last section. Then the absorption of this scalar creates a pair

of open strings on the D-string with energy $\frac{2\pi N}{n_w L}$ each, and a certain polarisation. The absorption probability is directly proportional to the number of open strings that already inhabit the state of the left moving open string, through the Bose enhancement factor. But all the microstates that give the same mass and charges to the D-brane configuration do not have the same number of quanta inhabiting this particular open string state, and so they have different absorption cross sections for the chosen incoming quantum. Thus by patient experimentation, information on the actual microstate can be deduced from absorption/scattering processes.

Since the black holes that correspond (in a different range of parameters) to this D-brane cluster have the same formula for the low energy cross section one may think that we can also find the microstate of a black hole by the same process. But here we make some observations that indicate that we have to be more careful before reaching such a conclusion.

Let us fix the charges and consider different ranges of g . For g too small, we presumably get thick solitonic strings and branes, much thicker than the Schwarzschild radius of the configuration. The case of g very large is just the S-dual of the elementary string with g small, so that we get solitonic 5-branes with an elementary string inside. This does not look like the black hole we want. Let us therefore take $g \sim 1$ in the discussion below.

In the D-brane calculation presented in the last section the absorbing levels were discrete, long lived levels. This was the case because we took a D-string that was wrapped on a very long circle, much longer than $g^{-1}L^{(S)}$. But if we wish to go towards the black hole limit of D-branes, we need to take a scale of compactification that is not too large for a given number of branes.

But as we reduce this length of the wrapping circle of the D-string below $g^{-1}L^{(S)}$, we saw in section 2 that the spectrum of excitations at low energies changes to one that has no sharp levels, just broad resonances at best, with the width of the resonances comparable to the height. The latter circumstance just means that the lifetime of the excited state is comparable to the time-scale $(\Delta E)^{-1}$, where ΔE is the typical separation between levels. This feature was a reflection of the fact that there is no scale in the 0-brane bound state spectrum (to which the D-string spectrum is dual when the compactification length is smaller than $g^{-1}L^{(S)}$) apart from the overall energy scale $(g')^{1/3}T^{(S)1/2}$.

Note that the above discussion is for a D-string in isolation and not inside a 5-D-brane. But let us assume that the above change of spectrum persists in the latter case as well. If we lose the picture of discrete levels before we reach the black hole limit of the D-brane

configuration, then it is not immediately clear what features can be picked up about black holes in scattering experiments. It is still true that with enough diligence we can extract all information from the D-brane system which is not in the black hole limit. But we see that the encoding of this information in the results of scattering experiments begins to change as we approach the black hole limit.

4.4. Low energy absorption cross-sections

What significance should we attach to the agreement between the absorption cross sections of D-branes and of black holes? This agreement has been demonstrated at low energies and for black holes near the extremal limit in [12][13]. (The result of [27] extends this to situations further from extremality.) It is possible that what we are seeing here is a universal structure of low energy amplitudes in a supergravity theory, perhaps for states not too far from extremality.

For example the low velocity scattering of BPS monopoles is given by geodesics on moduli space, and moduli space is a purely BPS construct. Thus the physics of slightly non-BPS processes (that of slowly moving monopoles) is capable of being described by knowledge of only BPS structures. It is not clear if low energy massless quanta are similar to slow moving massive BPS states, in the sense that moduli space physics may capture their interactions.

Recently it has been shown that there is an interesting algebraic relation giving the three point couplings of BPS states in string theory [31]. It would be interesting if this could be connected to the processes involving the absorption of low energy massless particles by massive BPS states.

Most interesting of course is the possibility that the agreement described in section 3 extends to higher energy quanta, smaller compactification radii, and to black holes far from extremality. An important question in this regard would be to understand the structure of BPS bound states involving different branes, and the excitations around such bound states, as the coupling is taken from weak towards strong. These issues are under investigation.

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